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Development of a Model to Predict the Flexural Strength of Concrete Using SDA as partial Replacement for Fine Aggregate

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Abstract— Saw Dust Ash (SDA) despite being an industrial waste, has been a significant addition to concrete to achieve quality and cost control as well as promote sustainability. In this study, SDA served as a fifth ingredient of concrete blend as it replaced 5% of the fine aggregate (sand). The other four ingredients were cement, sand, granite, and water. Scheffe's simplex theory was used for five mix ratios in a {5,2} experimental design which resulted in additional ten mix ratios. For purposes of verification and testing, additional fifteen mix ratios were generated. The thirty concrete mix ratios were subjected to laboratory experiments to determine the 28 days flexural strengths. The results of the first fifteen flexural strengths were used for the calibration of the model constant coefficients, while those from the second fifteen were used for the model verification using Scheffe's simplex lattice design. A mathematical regression model was derived from the results, with which the flexural strengths were predicted. The derived model was subjected to a two-tailed t-test with 5% significance, which ascertained the model to be adequate with an R^2 value of 0.8333. The study revealed that SDA can replace 5% of fine aggregate and promote sustainability, without compromising the 28 days flexural strengths.

Index Terms— Flexural Strength of concrete, Saw Dust Ash, Scheffe's simplex lattice, Sustainability.

1 Introduction

Many researchers and professionals in the construction industry are gradually incorporating the use of industrial waste materials in concrete. Such industrial waste materials as fly ash, saw dust ash (SDA), rice husk ash, quarry dust, and palm kernel shell ash are in use for various purposes. They have been used to replace fractions of either cement or fine aggregates, while others have been used to stabilise sub-base materials for pavement construction.

Saw dust is an industrial waste or bye-product of saw mills produced after the wood has been sawn to shape in the saw mill, and comes out in powder form. It has been used in concrete construction for over 30 years [1]. When saw dust is subjected to fire, it burns to ashes. That ash is called Saw Dust Ash (SDA).

In this study, SDA was used to partially replace 5% of the fine aggregate. A mathematical model was derived using Scheffe's regression theory, with which the flexural strengths of concrete were predicted in a 5-component mix (water-cement ratio, cement, sand, SDA, and granite).

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2 Literature Review

Concrete plays the biggest material role in the construction industry [2]. Several authors have studied and determined various means of actualizing economic and environmental sustainability in the construction industry with respect to concrete. Also, [3] have described the production of cement as a major source of environmental degradation as about 400kg of CO_2 is being emitted for every 600kg of cement produced. They therefore replaced 10% of cement with SDA which did not negatively affect the chloride permeability and thaw resistance of the concrete, but decreased the drying shrinkage, and increased the water absorption. Similarly, [4] found that replacing 5 to 15% cement content with saw dust increased the mechanical properties for 28days curing period and beyond. It also decreased the weight and cost. [5] also carried out a study by adding quarry dust to bituminous concrete to investigate the tensile strength, resulting to the development of a model. However, fewer researchers such as, [6] have carried out research on replacement of fine aggregates with SDA. Their research findings revealed that 10% replacement of fine aggregate with SDA will result in acceptable tensile, flexural, and compressive strengths as well as reduce the amount of wastes in the environment.

SDA has different particles that are mostly angular in shape. According to [6] SDA has a specific gravity of 2.5, fineness modulus of 1.78, water absorption of 0.56%, and bulk dry density of 1300kg/m^3 as against sand with specific gravity of 2.65, fineness modulus of 2.21, water absorption of 0.45%, and bulk dry density of 1512 kg/m^3. When 10% of SDA is added to the sand, these properties became 2.67, 2.2, 0.5%, and 1436kg/m^3 for specific gravity, fineness modulus, water absorption, and bulk dry density respectively. This is a significant indication that the mixture of sand and 10% SDA replacement gave similar physical properties with the 0% SDA replacement, making the mixture adequate for a fine aggregate. However, [7] SDA had a specific gravity of 2.19, bulk dry density of 1040kg/m³, and moisture content of 0.3%. This gives a bigger difference in the specific gravity of SDA as compared to that of sand. Furthermore, [8] shows that 50% of the SDA grain size is passing the AASHTO sieve no. 200 (75µm) while 31% is retained by sieve no. 325 (45 µm). This according to [8] justifies the fineness of SDA.

SDA, like many other concrete construction materials, contains several chemical compounds. According to [6] SDA has the following chemical composition by mass: 65.3% SiO_2, 4% Al_2O_3, 2.23% Fe_2O_3, 9.6% CaO, 5.8% MgO, 0.01% MnO, 0.07% Na_2O, 0.11% K_2O, 0.43% P_2O_5, and 0.45% SO_2. Summing up SiO_2, Al_2O_3, and Fe_2O_3 gives 71.53%. Similar work carried out by [9] reveals 67.95% SiO_2, 4.29% Al_2O_3, 2.15% Fe_2O_3, 9.47% CaO, 5.84% MgO, 0.01% MnO, 0.06% Na_2O, 0.11% K_2O, and 0.56% SO_3. Summing up SiO_2, Al_2O_3, and Fe_2O_3 gives 74.39. These, in accordance with [10] indicate that SDA is a good pozzolanic material. The chemical compositions as found by [6], [7], [9] all show that SDA has a high percentage of SiO_2 and small percentages of Al_2O_3 and Fe_2O_3, which are similar to those of sand with high percentage of about 95% SiO_2. Hence SDA can be used with sand as fine aggregate.

2.1 Scheffe's Simplex Theory

Several authors such as [11], [12], [13], [14], [5], [15], [16], [17], [18] have carried out concrete mixture research with the development of mathematical models. Most of such works were based on Scheffe's Simplex theory.

Scheffe's model is based on the simplex lattice and simplex theory or approach [19]. The simplex approach considers a number of components, *q*, and a degree of polynomial, *m*. The sum of all the i^{th} components is not greater than 1. Hence,

$$\sum_{i=1}^{q} x_i = 1 \quad (1)$$

$$x_1 + x_2 + \cdots + x_q = 1 \quad (2)$$

with $0 \le x \le 1$. The factor space becomes S_{q-1}. According to [19] the {*q*,*m*} simplex lattice design is a symmetrical arrangement of points within the experimental region in a suitable polynomial equation representing the response surface in the simplex region.

The number of points $C_m^{(q+m-1)}$ has (m+1) equally spaced values of x_i = 0, 1/m, 2/m, …. m/m. For a 3-component mixture with degree of polynomial 2, the corresponding number of points will be $C_2^{(3+2-1)}$ which gives 6 (eq. 3 or eq. 4 below) with number of spaced values, 2+1 = 3, that is x_i = 0, 1/2, and 1 and design points of (1,0,0), (0,1,0), (0,0,1), (1/2,1/2,0), (1/2,01/2), and (0,1/2,1/2). Similarly, for a {*5,2*} simplex, there will be 15 points with x_i = 0, 1/2, and 1 as spaced values. The 15 design points are (1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), (0,0,0,0,1), (1/2,1/2,0,0,0), (1/2,0,1/2,0,0), (1/2,0,0,1/2,0), (1/2,0,0,0,1/2), (0,1/2,1/2,0,0), (0,0,1/2,1/2,0), (0,0,0,1/2,1/2), (0,1/2,0,1/2,0), (0,0,1/2,0,1/2), (0,1/2,0,0,1/2).

$$N = C_n^{(q+n-1)} \quad (3)$$

or

$$N = \frac{(q+n-1)!}{(q-1)!\,(n)!} \quad (4)$$

For a polynomial of degree m with q component variables where eq. (2) holds, the general form is:

$$Y = b_0 + \sum b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ijk} x_i x_j x_k + \cdots + \sum b_{i1,i2\ldots in} x_{i1} x_{i2} x_{in} \quad (5)$$

Where 1 ≤ i ≤ q, 1 ≤ i ≤ j ≤ q, 1 ≤ i ≤ j ≤ k ≤ q, and b0 is the constant coefficient.

x is the pseudo component for constituents i, j, and k.

When {q,m} = {5,2}, eq. (5) becomes:

$$Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{15}x_1x_5 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{25}x_2x_5 + b_{34}x_3x_4 + b_{35}x_3x_5 + b_{45}x_4x_5 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{44}x_4^2 + b_{55}x_5^2 \quad (6)$$

and eq. (2) becomes

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1 \quad (7)$$

Multiplying eq. (7) by b_0 gives

$$b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 = b_0 \quad (8)$$

Multiplying eq. (7) successively by x1, x2, x3, x4, and x5 and making x1, x2, x3, x4, and x5 the subjects of the respective formulas:

$$\left.\begin{aligned} x_1^2 &= x_1 - x_1x_2 - x_1x_3 - x_1x_4 - x_1x_5 \\ x_2^2 &= x_2 - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5 \\ x_3^2 &= x_3 - x_1x_3 - x_2x_3 - x_3x_4 - x_3x_5 \\ x_4^2 &= x_4 - x_1x_4 - x_2x_4 - x_3x_4 - x_4x_5 \\ x_5^2 &= x_5 - x_1x_5 - x_2x_5 - x_3x_5 - x_4x_5 \end{aligned}\right\} \quad (9)$$

Substituting eq. (8) and eq. (9) into eq. (6) we have:

$$Y = b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{15}x_1x_5 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{25}x_2x_5 + b_{34}x_3x_4 + b_{35}x_3x_5 + b_{45}x_4x_5 + b_{11}(x_1 - x_1x_2 - x_1x_3 - x_1x_4 - x_1x_5) + b_{22}(x_2 - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5) + b_{33}(x_3 - x_1x_3 - x_2x_3 - x_3x_4 - x_3x_5) + b_{44}(x_4 - x_1x_4 - x_2x_4 - x_3x_4 - x_4x_5) + b_{55}(x_5 - x_1x_5 - x_2x_5 - x_3x_5 - x_4x_5)$$

$$Y = (b_0 + b_1 + b_{11})x_1 + (b_0 + b_2 + b_{22})x_2 + (b_0 + b_3 + b_{33})x_3 + (b_0 + b_4 + b_{44})x_4 + (b_0 + b_5 + b_{55})x_5 + (b_{12} - b_{11} - b_{22})x_1x_2 + (b_{13} - b_{11} - b_{33})x_1x_3 + (b_{14} - b_{11} - b_{44})x_1x_4 + (b_{15} - b_{11} - b_{55})x_1x_5 + (b_{23} - b_{22} - b_{33})x_2x_3 + (b_{24} - b_{22} - b_{44})x_2x_4 + (b_{25} - b_{22} - b_{55})x_2x_5 + (b_{34} - b_{33} - b_{44})x_3x_4 + (b_{35} - b_{33} - b_{55})x_3x_5 + (b_{45} - b_{44} - b_{55})x_4x_5 \quad (10)$$

Let

$$\left.\begin{aligned}
\beta_1 &= b_0 + b_1 + b_{11}\\
\beta_2 &= b_0 + b_2 + b_{22}\\
\beta_3 &= b_0 + b_3 + b_{33}\\
\beta_4 &= b_0 + b_4 + b_{44}\\
\beta_5 &= b_0 + b_5 + b_{55}\\
\beta_{12} &= b_{12} - b_{11} - b_{22}\\
\beta_{13} &= b_{13} - b_{11} - b_{33}\\
\beta_{14} &= b_{14} - b_{11} - b_{44}\\
\beta_{15} &= b_{15} - b_{11} - b_{55}\\
\beta_{23} &= b_{23} - b_{22} - b_{33}\\
\beta_{24} &= b_{24} - b_{22} - b_{44}\\
\beta_{25} &= b_{25} - b_{22} - b_{55}\\
\beta_{34} &= b_{34} - b_{33} - b_{44}\\
\beta_{35} &= b_{35} - b_{33} - b_{55}\\
\beta_{45} &= b_{45} - b_{44} - b_{55}
\end{aligned}\right\} \quad (11)$$

Substituting eq. (11) into eq. (10) gives

$$Y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{14} x_1 x_4 + \beta_{15} x_1 x_5 + \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 + \beta_{25} x_2 x_5 + \beta_{34} x_3 x_4 + \beta_{35} x_3 x_5 + \beta_{45} x_4 x_5 \quad (12)$$

$$Y = \sum_{i=1}^{5} \beta_i x_i + \sum_{1 \le i \le j \le 5} \beta_{ij} x_i x_j \quad (13)$$

Where the response, Y is a dependent variable (flexural strength of concrete). Eq. (12) is the general equation for a {5,2} polynomial, and it has 15 terms, which conforms to Scheffe's theory in eq. (3).

Let Y_i denote response to pure components, and Y_{ij} denote response to mixture components in i and j. If x_i =1 and x_j = 0, sice $j \neq i$, then

$$Y_i = \beta_i \quad (14)$$

Which means

$$\sum_{i=1}^{5} \beta_i x_i = \sum_{i=1}^{5} Y_i x_i \quad (15)$$

Hence, from eq. (14)

$$\left.\begin{aligned}
Y_1 &= \beta_1\\
Y_2 &= \beta_2\\
Y_3 &= \beta_3\\
Y_4 &= \beta_4\\
Y_5 &= \beta_5
\end{aligned}\right\} \quad (16)$$

According to [19],

$$\beta_{ij} = 4Y_{ij} - 2\beta_i - 2\beta_j \quad (17)$$

Substituting eq. (14)

$$\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \quad (18)$$

3 Materials and Methods

Water, cement, sand, SDA, and granite were the materials used to produce the concrete.

The first five concrete mix ratios derived from different mix design methods given as

BRE 12 = [0.54 1 1.9475 0.1025 2.95];
BRE 22 = [0.58 1 2.1185 0.1115 3.21];
USBR 22 = [0.58 1 2.2515 0.1185 3.29];
BIS 12 = [0.43 1 1.2065 0.0635 2.88];
ACI 12 = [0.55 1 1.8335 0.0965 3.09]

These can be put in matrix form as follows:

$$S = \begin{bmatrix} 0.54 & 0.58 & 0.58 & 0.43 & 0.55\\ 1 & 1 & 1 & 1 & 1\\ 1.9475 & 2.1185 & 2.2515 & 1.2065 & 1.8335\\ 0.1025 & 0.1115 & 0.1185 & 0.0635 & 0.0965\\ 2.95 & 3.21 & 3.29 & 2.88 & 3.09 \end{bmatrix} \quad (19)$$

Their corresponding pseudo components are given as:

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (20)$$

With centre points

X_{12} = [0.5 0.5 0 0 0]; X_{13} = [0.5 0 0.5 0 0];
X_{14} = [0.5 0 0 0.5 0]; X_{15} = [0.5 0 0 0 0.5];
X_{23} = [0 0.5 0.5 0 0]; X_{24} = [0 0.5 0 0.5 0];
X_{25} = [0 0.5 0 0 0.5]; X_{34} = [0 0 0.5 0.5 0];
X_{35} = [0 0 0.5 0 0.5]; X_{45} = [0 0 0 0.5 0.5]

According to,

Sij = XSi (21)

Substituting,

Substituting,

$$\begin{bmatrix} S_{12}\\ S_{13}\\ S_{14}\\ S_{15}\\ S_{23} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0\\ 0.5 & 0 & 0.5 & 0 & 0\\ 0.5 & 0 & 0 & 0.5 & 0\\ 0.5 & 0 & 0 & 0 & 0.5\\ 0 & 0.5 & 0.5 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0.54\\ 0.58\\ 0.58\\ 0.43\\ 0.55 \end{bmatrix} \quad (22)$$

This process is repeated for S24, S25, S34, S35, and S45. Similarly, this process is repeated for an additional 15 (control) points that will be used for the verification of the formulated model. The regular pentagons for the actual components with their corresponding pseudo components are given in figures (1) and (2) respectively.

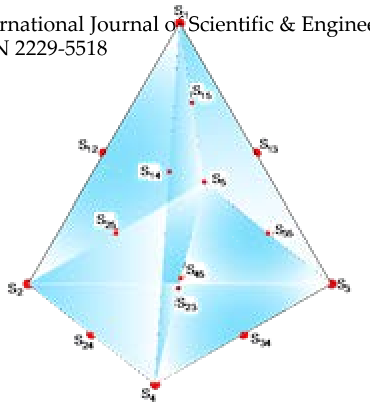

Fig. 1. Simplex plot for actual components

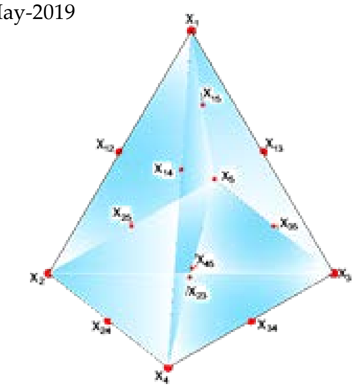

Fig. 2. Simplex plot for pseudo components

TABLE 1

MODEL MIX RATIOS

Sample Points	Actual Components					Response Y_{exp}	Pseudo Components				
	w-c ratio	Cement	Sand	SDA	Granite		w-c ratio	Cement	Sand	SDA	Granite
	S_1	S_2	S_3	S_4	S_5		X_1	X_2	X_3	X_4	X_5
BRE12	0.54	1	1.9475	0.1025	2.95	Y_1	1	0	0	0	0
BRE22	0.58	1	2.1185	0.1115	3.21	Y_2	0	1	0	0	0
USBR22	0.58	1	2.2515	0.1185	3.29	Y_3	0	0	1	0	0
BIS12	0.43	1	1.2065	0.0635	2.88	Y_4	0	0	0	1	0
ACI12	0.55	1	1.8335	0.0965	3.09	Y_5	0	0	0	0	1
N1	0.56	1	2.033	0.107	3.08	Y_{12}	0.5	0.5	0	0	0
N2	0.56	1	2.0995	0.1105	3.12	Y_{13}	0.5	0	0.5	0	0
N3	0.485	1	1.577	0.083	2.915	Y_{14}	0.5	0	0	0.5	0
N4	0.545	1	1.8905	0.0995	3.02	Y_{15}	0.5	0	0	0	0.5
N5	0.58	1	2.185	0.115	3.25	Y_{23}	0	0.5	0.5	0	0
N6	0.505	1	1.6625	0.0875	3.045	Y_{24}	0	0.5	0	0.5	0
N7	0.565	1	1.976	0.104	3.15	Y_{25}	0	0.5	0	0	0.5
N8	0.505	1	1.729	0.091	3.085	Y_{34}	0	0	0.5	0.5	0
N9	0.565	1	2.0425	0.1075	3.19	Y_{35}	0	0	0.5	0	0.5
N10	0.49	1	1.52	0.08	2.985	Y_{45}	0	0	0	0.5	0.5

TABLE 2

CONTROL POINTS

Sample Points	Actual Components					Response Y_{exp}	Pseudo Components				
	w-c ratio	Cement	Sand	SDA	Granite		w-c ratio	Cement	Sand	SDA	Granite
	S_1	S_2	S_3	S_4	S_5		X_1	X_2	X_3	X_4	X_5
C1	0.558	1	2.0463	0.1077	3.114	Y_{C1}	0.4	0	0.4	0	0.2
C2	0.52	1	1.7537	0.0923	3.078	Y_{C2}	0	0.6	0	0.4	0
C3	0.548	1	2.0083	0.1057	3.018	Y_{C3}	0.8	0	0.2	0	0
C4	0.49	1	1.5713	0.0827	3.012	Y_{C4}	0	0.4	0	0.6	0
C5	0.544	1	1.9019	0.1001	3.006	Y_{C5}	0.6	0	0	0	0.4
C6	0.55	1	2.0425	0.1075	3.208	Y_{C6}	0	0	0.8	0.2	0
C7	0.55	1	1.9589	0.1031	3.03	Y_{C7}	0.6	0.2	0	0	0.2
C8	0.514	1	1.6967	0.0893	3.054	Y_{C8}	0	0.4	0	0.4	0.2
C9	0.548	1	1.8563	0.0977	3.062	Y_{C9}	0.2	0	0	0	0.8
C10	0.46	1	1.4155	0.0745	2.962	Y_{C10}	0	0	0.2	0.8	0
C11	0.566	1	2.1071	0.1109	3.182	Y_{C11}	0.2	0	0.6	0	0.2
C12	0.544	1	1.9323	0.1017	3.152	Y_{C12}	0	0.2	0.4	0.2	0.2
C13	0.58	1	2.1451	0.1129	3.226	Y_{C13}	0	0.8	0.2	0	0
C14	0.532	1	1.7651	0.0929	3.072	Y_{C14}	0	0.2	0	0.2	0.6
C15	0.536	1	1.8715	0.0985	3.084	Y_{C15}	0.2	0.2	0.2	0.2	0.2

3.1 Flexural Strength test

Concrete samples were prepared for the flexural test to determine their tensile strengths. Prismatic beams of 150mmX150mmX450mm were formed. The third-point loading type was used to determine the modulus of rupture or flexural strength. The flexural strengths were then determined by,

$$f_b = \frac{3Pl}{2bd^2} \quad (23)$$

Where P = the total applied load (KN)
b and d = the breadth and depth of the flexural beam respectively in millimetres
l = the span length of beam in millimetres
Two replicates were made, and the average taken and recorded.

4 Results and Discussion

The results of the flexural strengths are presented in the table below.

TABLE 3

FLEXURAL STRENGTH OF CONCRETE

Sample	Curing	Load (KN)		$\frac{3L}{2bd^2}$	Flexural Strength (N/mm²)		
		A	B		A	B	Average
BRE12	7 Days	16.05	17.39	200	3.210	3.478	3.344
	28 Days	18.93	18.93	200	3.786	3.786	3.786
BRE22	7 Days	16.40	17.19	200	3.280	3.438	3.359
	28 Days	17.07	17.55	200	3.414	3.510	3.462
USBR22	7 Days	14.72	14.45	200	2.944	2.890	2.917
	28 Days	18.17	18.21	200	3.634	3.642	3.638
BIS12	7 Days	19.04	19.04	200	3.808	3.808	3.808
	28 Days	21.37	21.23	200	4.274	4.246	4.260
ACI12	7 Days	18.41	17.82	200	3.682	3.564	3.623
	28 Days	19.94	20.07	200	3.988	4.014	4.001
N1	7 Days	18.35	17.39	200	3.670	3.478	3.574
	28 Days	19.23	18.95	200	3.846	3.790	3.818
N2	7 Days	15.52	15.70	200	3.104	3.140	3.122
	28 Days	16.63	16.83	200	3.326	3.366	3.346
N3	7 Days	15.88	17.07	200	3.176	3.414	3.295
	28 Days	18.18	18.33	200	3.636	3.666	3.651
N4	7 Days	16.60	16.70	200	3.320	3.340	3.330
	28 Days	18.33	19.36	200	3.666	3.872	3.769
N5	7 Days	16.40	16.90	200	3.280	3.380	3.330
	28 Days	19.24	18.70	200	3.848	3.740	3.794
N6	7 Days	15.36	17.16	200	3.072	3.432	3.252
	28 Days	19.11	19.27	200	3.822	3.854	3.838
N7	7 Days	16.03	17.57	200	3.206	3.514	3.360
	28 Days	17.68	19.14	200	3.536	3.828	3.682
N8	7 Days	17.38	16.95	200	3.476	3.390	3.433
	28 Days	19.77	19.34	200	3.954	3.868	3.911
N9	7 Days	15.32	17.12	200	3.064	3.424	3.244
	28 Days	16.98	17.62	200	3.396	3.524	3.460
N10	7 Days	15.73	16.93	200	3.146	3.386	3.266
	28 Days	18.86	18.77	200	3.772	3.754	3.763

Sample	Curing	Load (KN)		$\frac{3L}{2bd^2}$	Flexural Strength (N/mm²)		
		A	B		A	B	Average
C1	7 Days	16.00	16.28	200	3.200	3.256	3.228
	28 Days	17.00	17.46	200	3.400	3.492	3.446
C2	7 Days	15.80	16.11	200	3.160	3.222	3.191
	28 Days	19.60	19.77	200	3.920	3.954	3.937
C3	7 Days	16.48	16.63	200	3.296	3.326	3.311
	28 Days	17.64	17.59	200	3.528	3.518	3.523
C4	7 Days	16.50	16.33	200	3.300	3.266	3.283
	28 Days	19.80	20.10	200	3.960	4.020	3.990
C5	7 Days	16.75	16.73	200	3.350	3.346	3.348
	28 Days	19.08	18.92	200	3.816	3.784	3.800
C6	7 Days	15.25	15.39	200	3.050	3.078	3.064
	28 Days	18.68	18.65	200	3.736	3.730	3.733
C7	7 Days	17.50	17.48	200	3.500	3.496	3.498
	28 Days	18.78	19.01	200	3.756	3.802	3.779
C8	7 Days	15.82	15.85	200	3.164	3.170	3.167
	28 Days	19.20	19.19	200	3.840	3.838	3.839
C9	7 Days	16.73	16.80	200	3.346	3.360	3.353
	28 Days	18.96	18.51	200	3.792	3.702	3.747
C10	7 Days	17.74	17.98	200	3.548	3.596	3.572
	28 Days	20.01	20.03	200	4.002	4.006	4.004
C11	7 Days	15.73	15.55	200	3.146	3.110	3.128
	28 Days	17.10	16.98	200	3.420	3.396	3.408
C12	7 Days	16.31	16.70	200	3.262	3.340	3.301
	28 Days	18.71	18.83	200	3.742	3.766	3.754
C13	7 Days	17.30	16.59	200	3.460	3.318	3.389
	28 Days	18.15	17.85	200	3.630	3.570	3.600
C14	7 Days	16.17	16.19	200	3.234	3.238	3.236
	28 Days	18.97	18.77	200	3.794	3.754	3.774
C15	7 Days	16.13	16.60	200	3.226	3.320	3.273
	28 Days	18.30	18.34	200	3.660	3.668	3.664

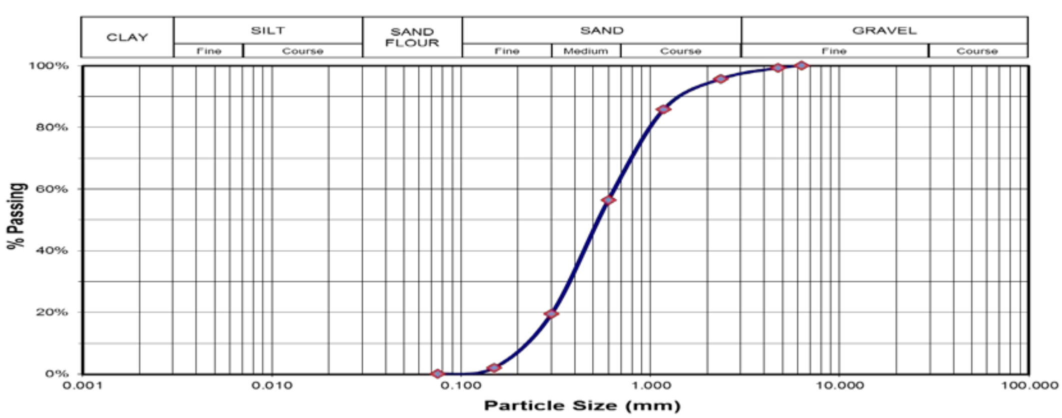

Fig. 3. Particle Size Distribution for Fine Aggregate with 5% SDA replacement

4.1 Scheffe's Model for 28 days Flexural Strength

The coefficients of polynomial from table (4), eq. (6), and eq. (18) are:
$\beta_1 = 3.786$, $\beta_2 = 3.462$, $\beta_3 = 3.638$, $\beta_4 = 4.26$, $\beta_5 = 4.001$, $\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2$, $\beta_{12} = 4 * 3.818 - 2 * 3.786 - 2 * 3.462 = 0.776$
Similarly, $\beta_{13} = -1.464$, $\beta_{14} = -1.488$, $\beta_{15} = -0.498$, $\beta_{23} = 0.976$, $\beta_{24} = -0.092$, $\beta_{25} = -0.198$, $\beta_{34} = -0.152$, $\beta_{35} = -1.438$, $\beta_{45} = -4.17$.
Substituting the above coefficients into eq. (12) gives

$$Y = 3.786x_1 + 3.462x_2 + 3.6384x_3 + 4.26x_4 + 4.001x_5 + 0.776x_1x_2 - 1.464x_1x_3 - 1.488x_1x_4 - 0.498x_1x_5 + 0.976x_2x_3 - 0.092x_2x_4 - 0.198x_2x_5 - 0.152x_3x_4 - 1.438x_3x_5 - 4.17x_4x_5 \quad (24)$$

Eq. (24) above is the mathematical model to predict the 28 days flexural strength of concrete using SDA to replace 5% of fine aggregate.

TABLE 4
EXPERIMENTAL AND PREDICTED VALUES OF 28 DAYS FLEXURAL STRENGTH OF CONCRETE

Sample Points	Response Y	Pseudo Components					Flexural strength	Flexural strength
		w-c ratio	Cement	Sand	SDA	Granite		
		X_1	X_2	X_3	X_4	X_5	Y_{exp}(N/mm^2)	Y_{pred}(N/mm^2)
BRE12	Y_1	1	0	0	0	0	3.786	3.786
BRE22	Y_2	0	1	0	0	0	3.462	3.462
USBR22	Y_3	0	0	1	0	0	3.638	3.638
BIS12	Y_4	0	0	0	1	0	4.26	4.26
ACI12	Y_5	0	0	0	0	1	4.001	4.001
N1	Y_{12}	0.5	0.5	0	0	0	3.818	3.818
N2	Y_{13}	0.5	0	0.5	0	0	3.346	3.346
N3	Y_{14}	0.5	0	0	0.5	0	3.651	3.651
N4	Y_{15}	0.5	0	0	0	0.5	3.769	3.769
N5	Y_{23}	0	0.5	0.5	0	0	3.794	3.794
N6	Y_{24}	0	0.5	0	0.5	0	3.838	3.838
N7	Y_{25}	0	0.5	0	0	0.5	3.682	3.682
N8	Y_{34}	0	0	0.5	0.5	0	3.911	3.911
N9	Y_{35}	0	0	0.5	0	0.5	3.46	3.46
N10	Y_{45}	0	0	0	0.5	0.5	3.763	3.763
C1	Y_{C1}	0.4	0	0.4	0	0.2	3.446	3.381
C2	Y_{C2}	0	0.6	0	0.4	0	3.937	3.759
C3	Y_{C3}	0.8	0	0.2	0	0	3.523	3.522
C4	Y_{C4}	0	0.4	0	0.6	0	3.99	3.919
C5	Y_{C5}	0.6	0	0	0	0.4	3.8	3.752
C6	Y_{C6}	0	0	0.8	0.2	0	3.733	3.738
C7	Y_{C7}	0.6	0.2	0	0	0.2	3.779	3.79
C8	Y_{C8}	0	0.4	0	0.4	0.2	3.839	3.741
C9	Y_{C9}	0.2	0	0	0	0.8	3.747	3.878
C10	Y_{C10}	0	0	0.2	0.8	0	4.004	4.111
C11	Y_{C11}	0.2	0	0.6	0	0.2	3.408	3.372
C12	Y_{C12}	0	0.2	0.4	0.2	0.2	3.754	3.68
C13	Y_{C13}	0	0.8	0.2	0	0	3.6	3.653
C14	Y_{C14}	0	0.2	0	0.2	0.6	3.774	3.741
C15	Y_{C15}	0.2	0.2	0.2	0.2	0.2	3.664	3.627

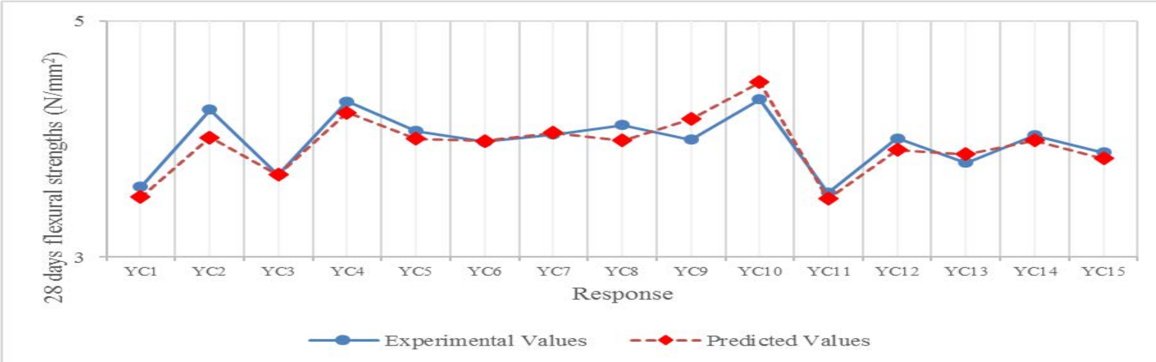

Fig. 4. Comparison between Experimental and Predicted 28days Flexural Strengths

4.2 Test of Adequacy of the Model

The coefficients of polynomial from table (4),
A two-tailed student t-test was carried out at 95% confidence level, which implies 100 – 95 = 5% significance. Since it is a two-tailed, significance = 5/2 = 2.5%
Hence significance level = 100 – 2.5 = 97.5%
Let D be difference between the experimental and predicted

The mean of the difference,

$$D_a = \frac{1}{n}\sum_{i=1}^{n} D_i \qquad (25)$$

The variance of the difference,

$$S^2 = \left(\frac{1}{n-1}\right)\sum_{i=1}^{n}(D-D_a)^2_i \qquad (26)$$

$$t_{calculated} = \frac{D_a\sqrt{n}}{S} \qquad (27)$$

Where n = number of observations with degree of freedom n – 1.

TABLE 5

STUDENT T-TEST FOR 28DAYS FLEXURAL STRENGTH OF CONCRETE

Sample	Curing	Flexural Strength (N/mm²)		t-test		
		$Y_{experimental}$	$Y_{predicted}$	$D=Y_{exp}-Y_{pred}$	D_a-D	$(D_a-D)^2$
C1	28 Days	3.446	3.381	0.065	-0.043	0.002
C2	28 Days	3.937	3.759	0.178	-0.156	0.024
C3	28 Days	3.523	3.522	0.001	0.021	0
C4	28 Days	3.99	3.919	0.071	-0.049	0.002
C5	28 Days	3.8	3.752	0.048	-0.026	0.001
C6	28 Days	3.733	3.738	-0.005	0.027	0.001
C7	28 Days	3.779	3.79	-0.011	0.033	0.001
C8	28 Days	3.839	3.741	0.098	-0.076	0.006
C9	28 Days	3.747	3.878	-0.131	0.153	0.023
C10	28 Days	4.004	4.111	-0.107	0.129	0.017
C11	28 Days	3.408	3.372	0.036	-0.014	0
C12	28 Days	3.754	3.68	0.074	-0.052	0.003
C13	28 Days	3.6	3.653	-0.053	0.075	0.006
C14	28 Days	3.774	3.741	0.033	-0.011	0
C15	28 Days	3.664	3.627	0.037	-0.015	0
TOTAL				0.332		0.086
AVERAGE D_a				0.022		

$$S^2 = \frac{0.086}{15-1} = 0.006$$

S = √0.006 = 0.078, $t_{calculated}$ = 1.099
From the t-table, $t_{(\beta,v)}$ can be determined where v = 15 – 1 = 14, and β = significance level.
$t_{(0.95,14)}$ = 2.145
Since $t_{calculated} < t_{(0.975,14)}$, and lies between -2.145 and 2.145, therefore there is no significant difference between the experimental and predicted responses, H_0 is accepted, and H_a is rejected. The model is confirmed to be adequate.

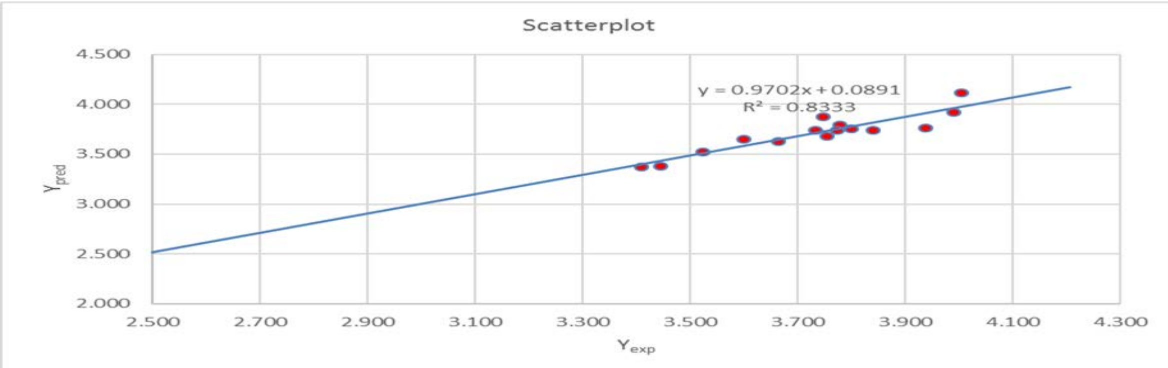

Fig. 5. Scatterplot of Predicted vs. Experimental 28days Flexural Strength

The R^2 value of 0.8333 indicates that the experimental results are highly correlated to the predicted results. This is also an indication that the model is fit and adequate.

4 Conclusion

After successfully replacing fine aggregate with 5% SDA, the 28 days flexural strengths are acceptable (between 3.3 and 4.3N/mm²). A regression model has been generated from the resulting experimental flexural strengths using Scheffe's simplex theory. A two-tailed t-test was carried out at 5% significance level, which confirmed the adequacy of the derived model with an R^2 value of 0.8333. The results also confirmed that SDA is a suitable material to replace a small fraction of fine aggregate in a bid to promote environmental sustainability.

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